Estimating RF Near Field Power Levels

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Estimating transmitted RF power near a microwave antenna is a common request. FCC OET Bulletin 65, a document from 1997, addresses this issue – so is there anything left to consider? Well, there are at least two issues. First, as illustrated below, Bulletin 65 is incorrect for the Part 101 parabolic antenna case. It also does not address the square panels used in Part 15 applications. First let's consider Bulletin 65 and the parabolic antenna case.

For microwave frequencies the FCC has regulatory limits[1] on RF exposure: 1 mw per cm^2 for uncontrolled areas and 5 mw per cm^2 for controlled areas. The Commission, in its OET Bulletin 65,[2] suggests that the transmitter power density for circular aperture ("parabolic") antennas may be estimated using the following equation:

$$S_{nf} = 16\ \eta\ P / [\ \pi\ D^2\ ]$$

S_{nf} = maximum near-field power density [mw/cm^2]
η = aperture efficiency (fraction, typically 0.5-0.75)
P = power fed to the antenna [mw]
D = antenna diameter [cm]

Let's consider a hypothetical 6-foot (182.88 cm) antenna with 55% (0.55 as a fraction) illumination efficiency. What would be the transmit power P that would result in a near field power density S_{nf} of 1 mw/cm^2?

$$P = \pi\ D^2 / [16\ \eta\ ] = \pi \times 182.88^2 / [\ 16 \times 0.55\ ] = 11{,}940 \text{ mw} = 40.8 \text{ dBm}$$

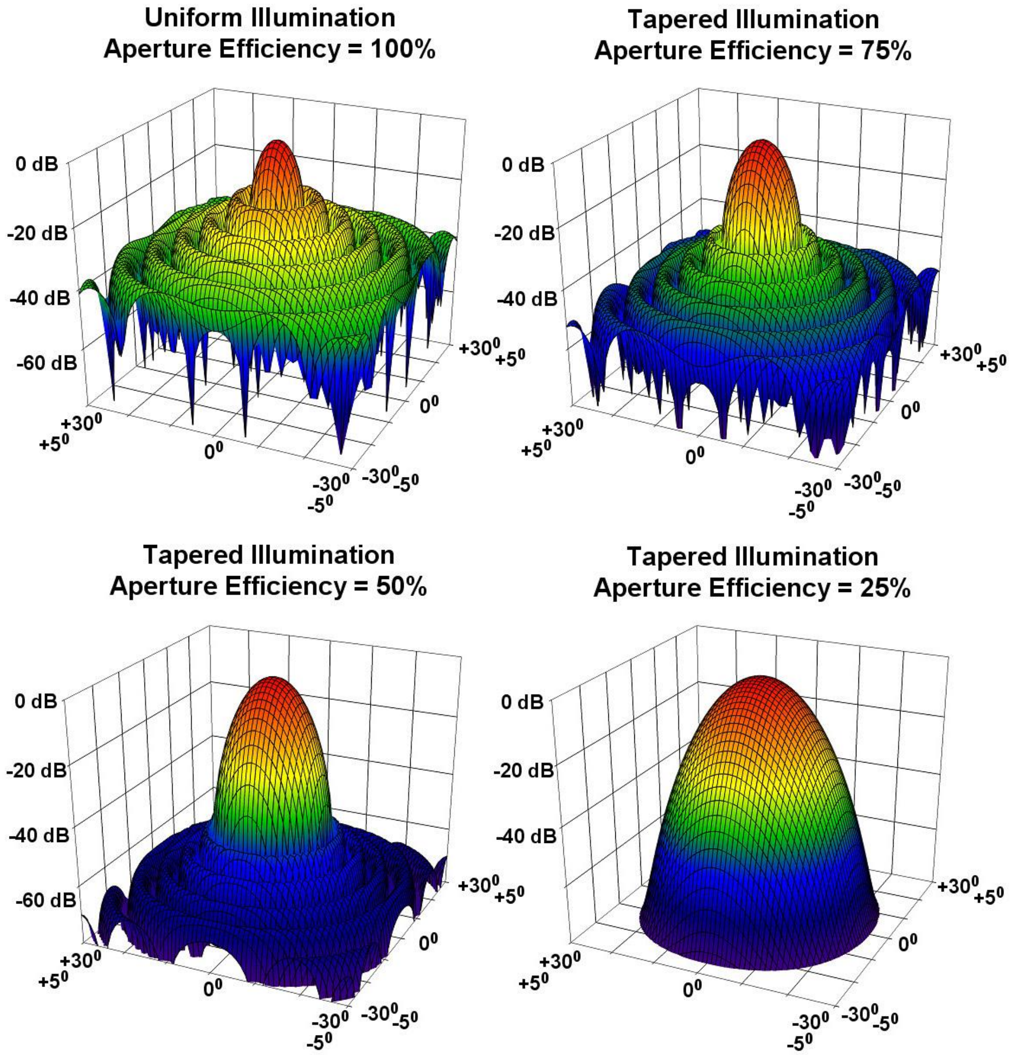
The maximum near-field power density is on a center line directly in front of the center of the antenna (boresight). The above equation is, on the face of it, contrary to physics. As antenna aperture illumination is tapered away from the edge of the (essentially lossless) antenna, the illumination efficiency of the antenna is reduced while the relative power is increased near the center of the antenna. This means the worst case near field density must INCREASE as aperture efficiency is reduced. The Bulletin 65 formula has this backwards. While it is correct for the 100% illumination case, from a physics perspective, it simply cannot be correct for lower illumination efficiencies.

Circular ("Parabolic") Aperture Antennas

The circular antenna achieves its gain by spreading the transmitted energy across the face of the antenna. Equal illumination of the face of the antenna results in the most

[1] CFR Title 47: Telecommunication §1.1310 (e) (1) Radiofrequency Radiation Exposure Limits
[2] *Evaluating Compliance with FCC Guidelines for Human Exposure to Radiofrequency Electromagnetic Fields*, OET Bulletin 65, Edition 97-01, August 1997, equation (13), page 28.

antenna gain. However, side lobe performance is quiet poor. For practical antennas, the transmitted power is spread across the face of the antenna with more toward the center and less toward the edge. The greater the taper, the lower the gain of the antenna (lower illumination efficiency) but the better the side lobe performance.



2 Foot Diameter, 5 GHz unlicensed band (5.5 GHz), D/λ = 11 (+30º to -30º)
10 Foot Diameter, Lower 6 GHz licensed band (6.2 GHz), D/λ = 66 (+5º to -5º))

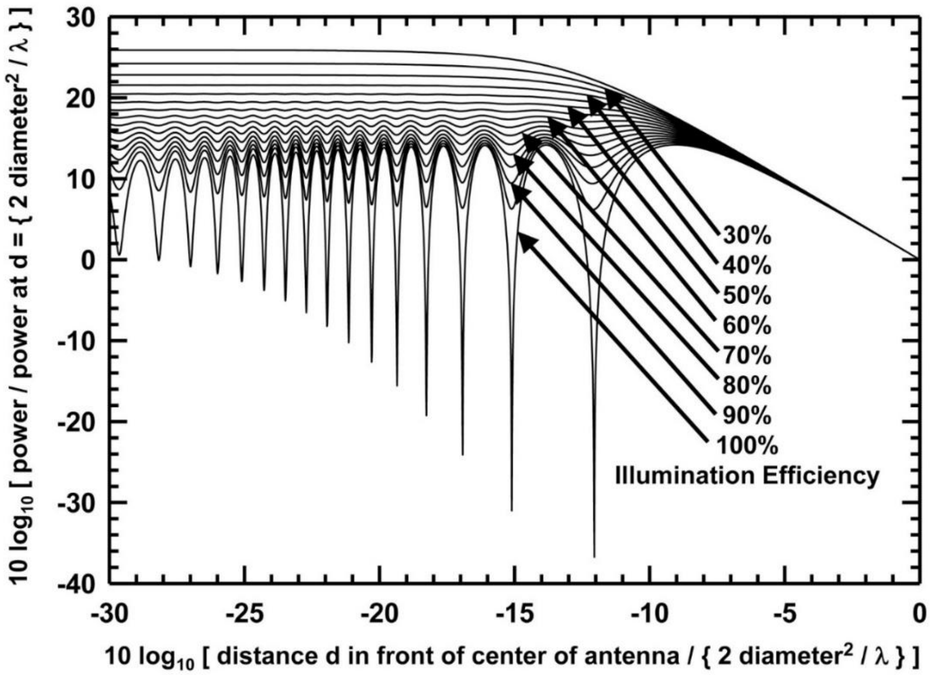
Circular Antenna Far Field Radiation Pattern

Antenna illumination efficiency may be estimated using the following formulas.

$10 \log (\eta) = -10.1 + G - 20 \log (F) - 20 \log (D)$

η = antenna illumination efficiency (fraction, 0 to 1) = $10^{[10 \log (\eta)/10]}$
G = antenna gain (dBi)
F = Operating Frequency (GHz)
D = antenna diameter (ft)

Kizer described the procedure for calculating near field power density for circular antennas[3]. Using his methods, we may calculate near field density referred to the far field crossover point d = 2 diameter^2 / free space wavelength (λ).



Circular Antenna Near Field Power Density

The above values were calculated for a 6-foot diameter circular antenna operating at 6.175 GHz. Other diameters or frequencies would have shifted the location of the nulls but otherwise the power density plot would have been unchanged.

Although the power density may change with distance for high illumination efficiencies, we are interested in estimating worst case near field power (worst case values on the far left of the above graph). For that, we may use the following approximations:

Relative Power $= A + B\,N + C\,N^2 + D\,N^3 + E\,N^4$

A = 37.71623065015471
B = - 0.6319006509486316
C = 0.007468595697388079
D = - 0.000049972215606888844
E = 0.00000014955941890925559

N = 100 * η = antenna illumination efficiency (%)

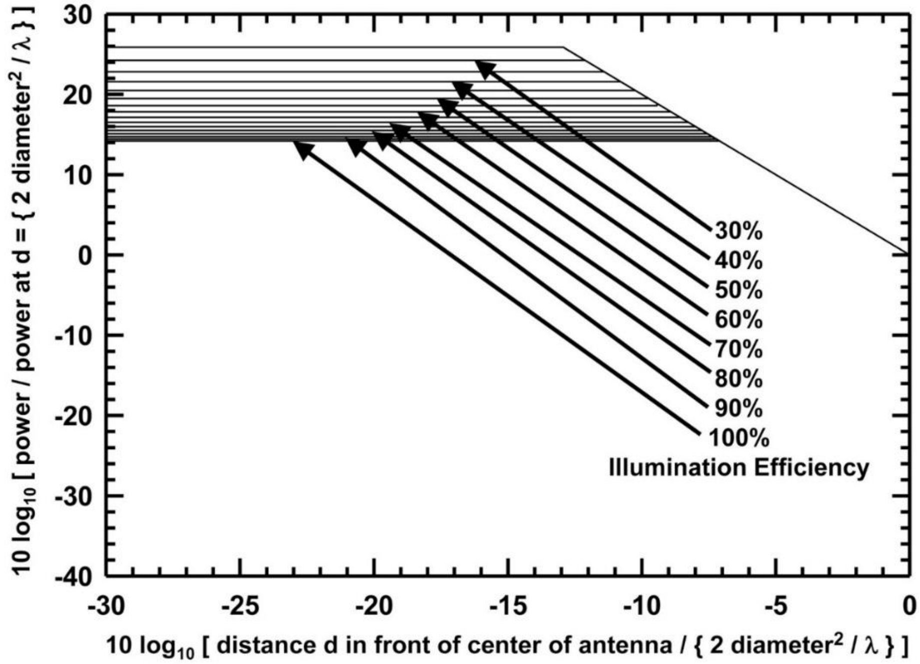
OR

[3] Kizer, *Digital Microwave Communication*, Wiley, 2013, Chapter 8, pages 264-274.

Relative Power = -2 X = 20 log [distance in front of antenna / 2 diameter^2 λ]

Use whichever Relative Power value is smaller where λ = [0.98357 / frequency (GHz)] is the free space wavelength.

These approximations are graphed below.



Worst Case Circular Antenna Near Field Power Density Approximation

Kizer derived the power density S_{nf} for the far field cross over distance[4] (0 on the X axis of the graph above).

S_{nf} (far field crossover) = π η P / [16 D^2], parameters as previously defined

Therefore

S_{nf} (power density very near the antenna center)
= S_{nf} (near antenna power / far field crossover power) S_{nf} (far field crossover)
= (relative power from graph) π η P / [16 D^2]
= [$10^{\text{graph value}/10}$] π η P / [16 D^2]

If transmit power P to achieve a power density S_{nf} is required, the formula becomes the following.

P (mw) = 16 D^2 S_{nf} / { [$10^{\text{graph value}/10}$] π η }

[4] Kizer, *Digital Microwave Communication*, Wiley, 2013, Chapter 8, pages 273-274.

$P\ (dBm) = 10 \log_{10} [P\ (mw)]$

All variables are as previously defined.

Let's consider our previous 6-foot (182.88 cm) diameter 55% illumination efficiency (η = 0.55) circular antenna. From our above formulas, the antenna in the limit will have a near field density 18.6 dB (power ratio = $10^{18.6/10}$ = 72.4) greater than at the far field crossover. Therefore the following relationships apply for a near field power density of S_{nf} (near antenna center) = 1 mw / cm^2.

$P = 16 \times 182.88^2 \times 1 / [\ 72.4 \times \pi \times 0.55\ ] = 4{,}278$ mw = 36.3 dBm

The FCC Bulletin 65 formula is too optimistic by 4.5 dB.

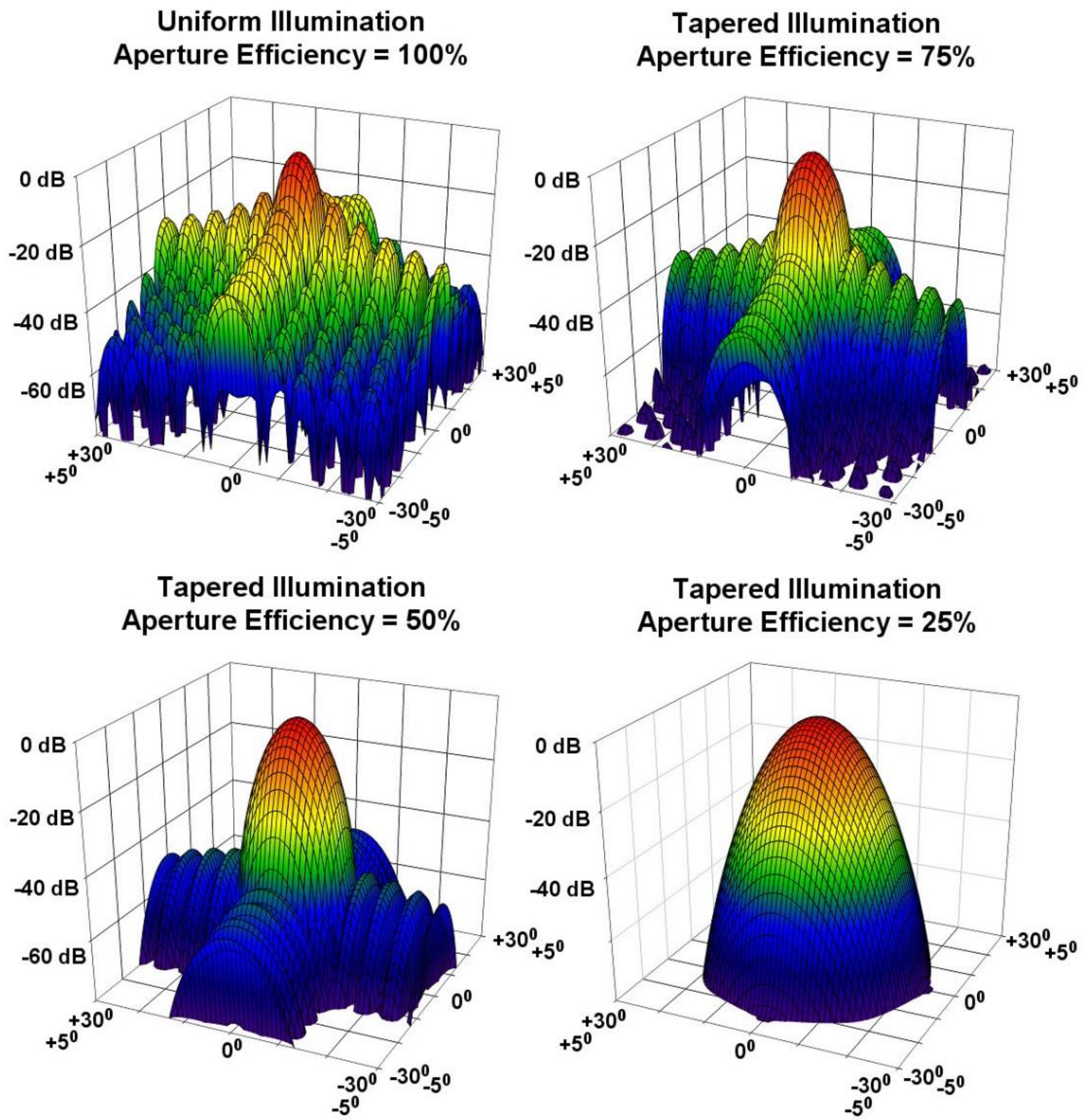
Using the formula $P = 16 \times D^2 \times 1 / [\ 72.4 \times \pi \times 0.55\ ]$ and the previous definitions, we may calculate the power delivered to a parabolic 55% efficient antenna which will just reach a 1 mw per cm^2 limit (uncontrolled areas).

Diameter (feet)	Parabolic Antenna (55% efficiency) Maximum Power (dBm)
15	44.3
12	42.3
10	40.7
8	38.8
6	36.3
4	32.8
3	30.3
2.6	29.0
2	26.8
1	20.7
0.5	14.7
0.25	8.7

For controlled areas, the above power values are increased 7 dB.

Square ("Panel") Aperture Antennas

Square antennas are commonly used in unlicensed bands. They are typically implements using patches on a dielectric material. Because of their dielectric losses, they are not used at frequencies higher than 6 GHz. Like circular antennas, aperture illumination affects antenna gain efficiency as well as side lobe performance. Since side lobe performance is also a function of antenna orientation, square antennas are often rotated to look like a diamond to minimize sidelobe radiation.

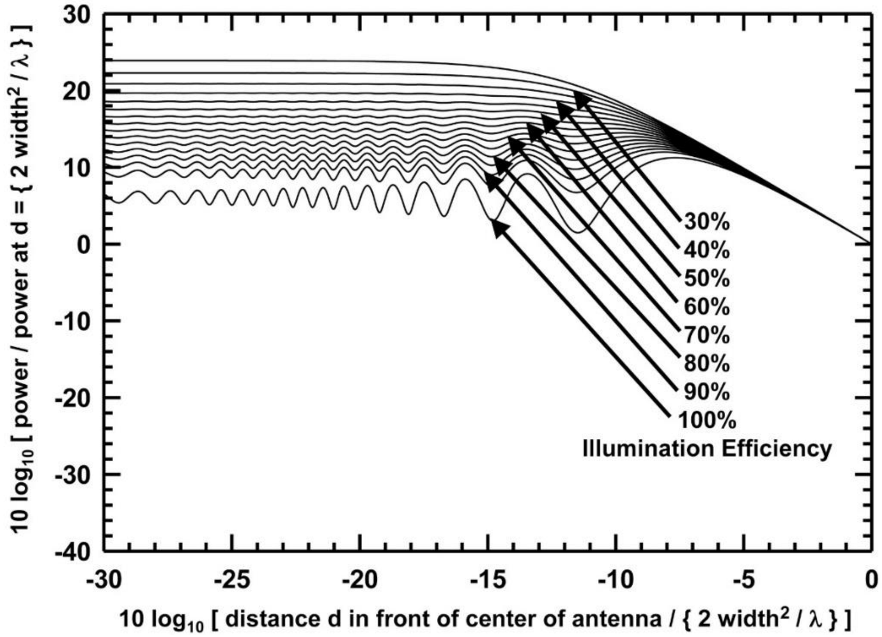


2 Foot Width, 5 GHz unlicensed band (5.5 GHz), D/λ = 11 (+30º to -30º)
10 Foot Width, Lower 6 GHz licensed band (6.2 GHz), D/λ = 66 (+5º to -5º))

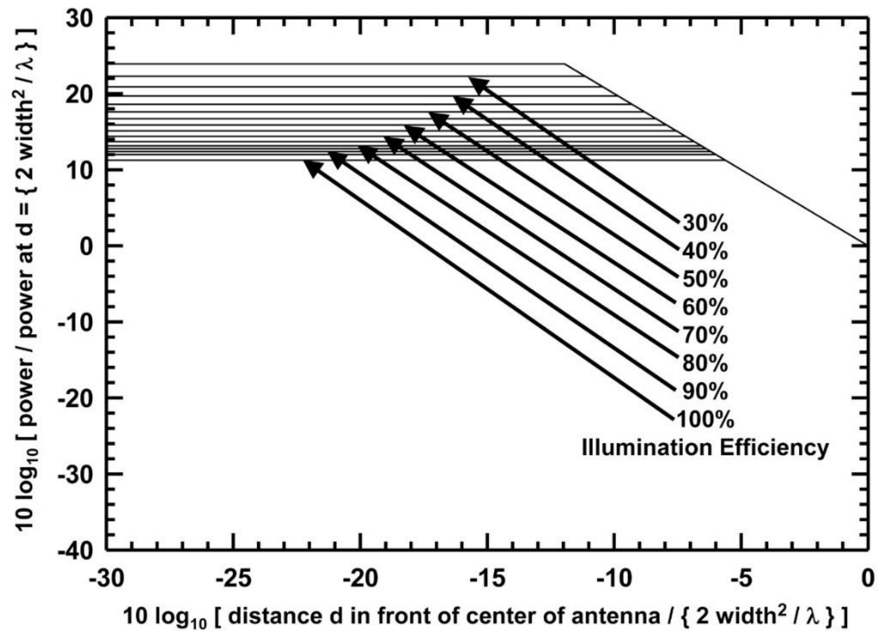
Square Antenna Far Field Radiation Pattern

Parabolic antennas are essentially lossless. Square antennas are relatively lossy. Using gain to estimate illumination efficiency will not be accurate. Illumination efficiency may be inferred from antenna beamwidth[5]. Using Kizer's methodology, we may calculate worst case near field power density as a function of antenna width. The following graph was calculated for a 6-foot square antenna operating at 6.175 GHz. Other frequencies and sizes would have shifted the location of the peaks and nulls but not changed their peak or null values.

[5] Kizer, *Digital Microwave Communication*, Wiley, 2013, Chapter 8, pages 277-278



Square Antenna Near Field Power Density



Worst Case Square Antenna Near Field Power Density Approximation

We may use the following approximations for worst case power density:

Relative Power $= A + B\,N + C\,N^2 + D\,N^3 + E\,N^4 + F\,N^5 + G\,N^6 + H\,N^7 + J\,N^8$

A = 62.59521011781284
B = - 4.763302760061157
C = 0.2755251943029563
D = - 0.009641587750450044

$E = \ 0.0002071182890199265$
$F = - 2.748139825835553E\text{-}06$
$G = \ 2.181156282683387E\text{-}08$
$H = - 9.405150951733821E\text{-}11$
$J = \ 1.674477996684913E\text{-}13$

$N = 100 * \eta$ = antenna illumination efficiency (%)

OR

Relative Power = $-2 X = 20 \log$ [distance in front of antenna / 2 width^2 λ]

Use whichever Relative Power value is smaller where λ = [0.98357 / frequency (GHz)] is the free space wavelength.

While square antenna efficiency could, in theory, be inferred from size, gain and frequency, most square antennas are small and relatively lossy. The efficiency of interest, η, is the radiation efficiency that is directly related to radiation pattern characteristics. Rather than use gain (which is affected by broad bandwidth dielectric losses), efficiency is more accurately estimated directly from the 3dB beam-width specification ϕ_{3dB} using the following formulas:

$\eta = (N0 + N1\,\beta + N2\,\beta^2 + N3\,\beta^3 + N4\,\beta^4) / (1 + D1\,\beta + D2\,\beta^2 + D3\,\beta^3 + D4\,\beta^4)$

$\beta = [W / \lambda] \sin [\phi_{3dB}/2]$

$1.49 \geq \beta \geq 0.447$

$0.1 \leq \eta \leq 1.0$

ϕ_{3dB} = antenna 3 dB beamwidth (angle between the two - 3 dB points relative to boresight gain)

N0 = - 0.4468979109577574
N1 = 2.705347403057084
N2 = - 5.689139811168476
N3 = 5.017375871680245
N4 = - 1.037085334383484
D1 = - 7.914244751535077
D2 = 24.1096714821637
D3 = - 33.58979930453501
D4 = 18.85685129777957

Square antennas are used in the unlicensed bands where gain is more important than sidelobe performance. Virtually all square antennas achieve 100% illumination efficiency. The calculated worst case near field power density is 11.3 dB above the near field crossover point.

Kizer derived the power density for the far field cross over distance[6] (0 on the X axis of the graph above).

S_{nf} (far field crossover) = η P / [4 W^2]

Where P and are as before and W is width in cm.

S_{nf} (power density very near the antenna center)

= S_{nf} (near antenna power / far field crossover power) S_{nf} (far field crossover)

= (relative power from graph) η P / [4 W^2]

= [$10^{\text{graph value/10}}$] η P / [4 W^2]

If transmit power P to achieve a power density S_{nf} is required, the formula becomes the following.

$$P\ (mw) = 4\ W^2\ S_{nf} / \{ [10^{\text{graph value/10}}]\ \eta \}$$

$$P\ (dBm) = 10 \log_{10} [P\ (mw)]$$

All variables are as previously defined.

Assuming 100% illumination efficiency (η =1.0), the power P applied to the antenna to achieve 1 mw per cm^2 would be found from the following.

$$P\ (mw) = 4\ W^2 / 10^{11.3/10} = 0.297\ W^2$$

Using this formula, we may calculate the power delivered to a square 100% illumination efficient antenna which will just reach a 1 mw per cm^2 limit (uncontrolled areas).

Width (feet)	Square Antenna (100% efficiency) Maximum Power (dBm)
15	48.0
12	46.0
10	44.4
8	42.5
6	40.0
4	36.5
3	34.0
2.6	32.7
2	30.5
1	24.4
0.5	18.4
0.25	12.4

[6] Kizer, *Digital Microwave Communication*, Wiley, 2013, Chapter 8, pages 288-289

For controlled areas, the above power values are increased 7 dB.

By the way, when evaluating the flat panel antennas, be sure to subtract the ohmic loss of the antenna from the antenna input power evaluation. This loss is usually on the order of 2 dB. It can be determined by comparing the rated boresight gain to the theoretical gain.

G(dBi) = 11.1 + 20 log f (GHz) + 10 log area (ft^2) + 10 log η = theoretical gain

η = aperture illumination efficiency (fraction, typically 1.0 for panel antennas)

APPENDIX

Additional Data

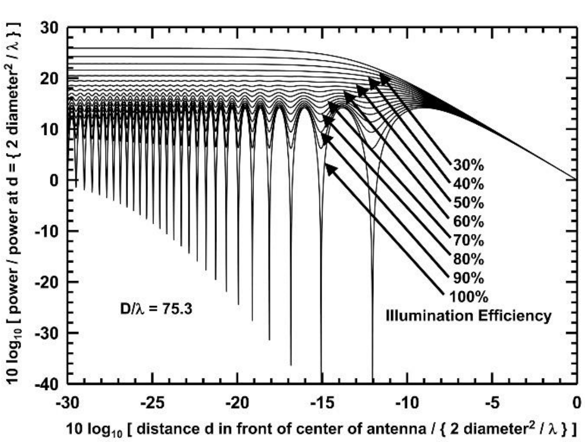
Calculated values used to derive the above results.

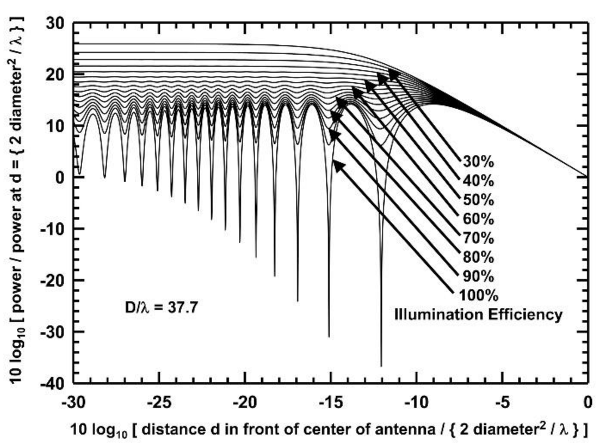
Circular ("Parabolic") Antennas

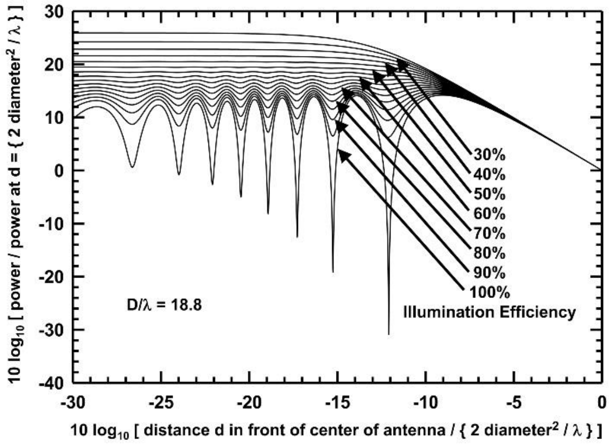
Maximum Power Density (dB relative to power density at [d / (2 D^2 / λ)] = 1)																
Diameter (ft)	η = 100 %	95%	90%	85%	80%	75%	70%	65%	60%	55%	50%	45%	40%	35%	30%	25%
12	14.2	14.4	14.8	15.1	15.5	16.0	16.5	17.2	17.9	18.6	19.5	20.5	21.6	22.8	24.2	25.9
6	14.2	14.4	14.7	15.1	15.5	16.0	16.5	17.1	17.8	18.6	19.5	20.5	21.6	22.8	24.2	25.9
3	14.2	14.4	14.7	15.1	15.5	16.0	16.5	17.1	17.8	18.6	20.5	20.5	21.6	22.8	24.2	25.9
2	14.2	14.4	14.7	15.0	15.5	15.9	16.5	17.1	17.8	18.6	19.5	20.5	21.6	22.8	24.3	25.9
1	14.1	14.4	14.6	15.0	15.3	15.9	16.5	17.1	17.8	18.7	19.6	20.6	21.7	22.9	24.4	26.1
0.5	13.8	14.3	14.6	15.0	15.4	15.8	16.4	17.2	18.0	18.8	19.8	20.9	22.1	23.5	25.1	26.9
0.25	13.0	14.0	14.6	15.2	15.9	16.7	17.6	18.4	19.2	20.1	20.9	21.8	22.8	23.8	25.0	26.3
0.125	11.64	13.44	14.21	14.85	15.43	15.98	16.52	17.08	17.65	18.26	18.91	19.62	20.41	21.29	22.31	23.53

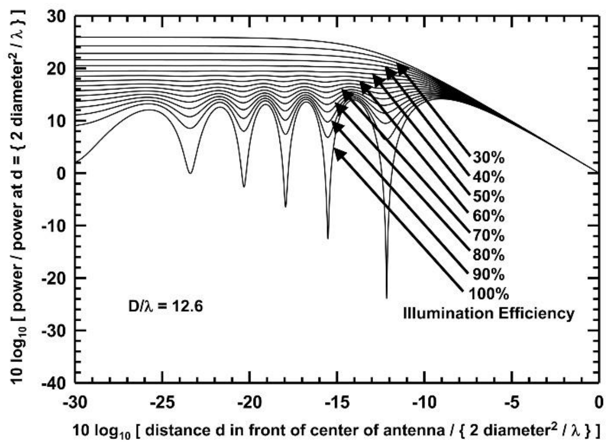
	Diameter (ft)					
D / λ	5.788 GHz	6.175 GHz	6.7 GHz	11.2 GHz	18.7 GHz	22.4 GHz
75.3	12.8	12.0	11.1	6.62	3.96	3.31
37.7	6.40	6.00	5.53	3.31	1.98	1.65
18.8	3.20	3.00	2.76	1.65	0.991	0.827
12.6	2.13	2.00	1.84	1.10	0.660	0.551
6.28	1.07	1.00	0.922	0.551	0.330	0.276
3.14	0.533	0.500	0.461	0.276	0.165	0.138
1.57	0.267	0.250	0.230	0.138	0.0826	0.0689
0.785	0.133	0.125	0.115	0.0689	0.0413	0.0345

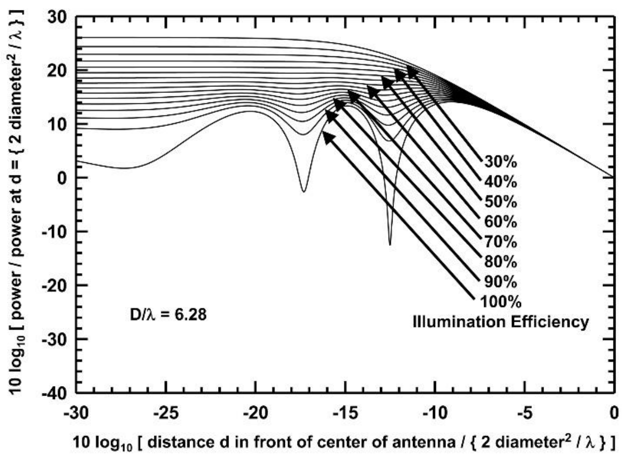
	2 D^2 / λ (ft)					
D / λ	5.788 GHz	6.175 GHz	6.7 GHz	11.2 GHz	18.7 GHz	22.4 GHz
75.3	1929	1808	1666	997	597	498
37.7	482	452	417	249	149	125
18.8	121	113	104	62.3	37.3	31.2
12.6	53.6	50.2	46.3	27.7	16.6	13.8
6.28	13.4	12.6	11.6	6.92	4.15	3.46
3.14	3.35	3.14	2.89	1.73	1.04	0.865
1.57	0.837	0.785	0.723	0.433	0.259	0.216
0.785	0.209	0.196	0.181	0.108	0.0648	0.0541

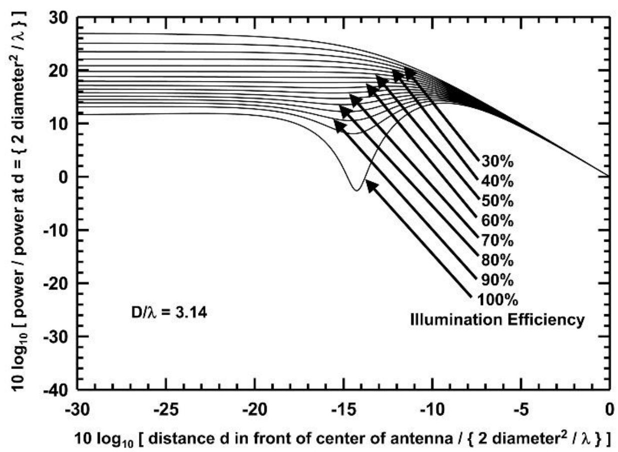


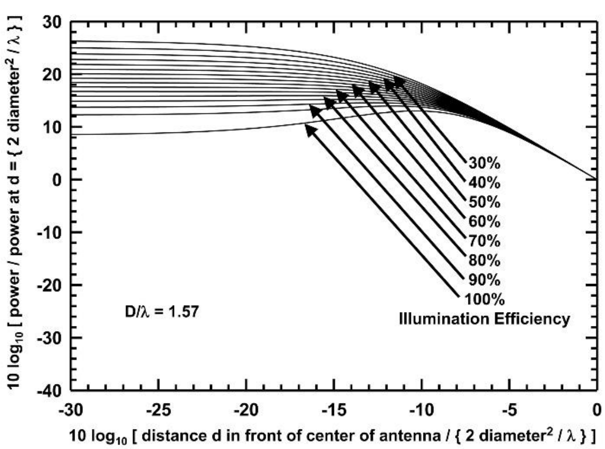


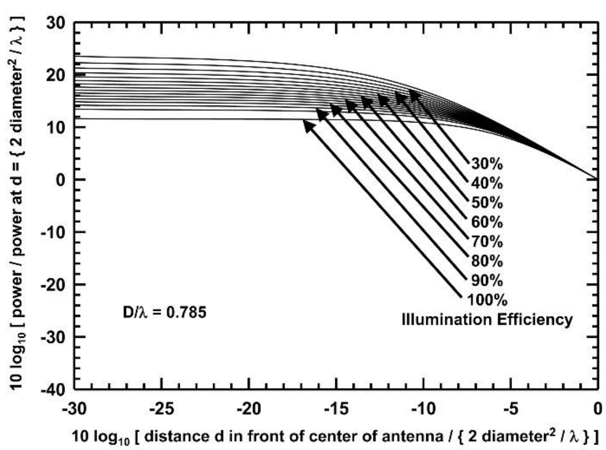

10 log10 [power / power at d = { 2 diameter2 / λ }]
D/λ = 18.8
30%
40%
50%
60%
70%
80%
90%
100%
Illumination Efficiency
10 log10 [distance d in front of center of antenna / { 2 diameter2 / λ }]


10 log10 [power / power at d = { 2 diameter2 / λ }]
D/λ = 12.6
30%
40%
50%
60%
70%
80%
90%
100%
Illumination Efficiency
10 log10 [distance d in front of center of antenna / { 2 diameter2 / λ }]


10 log10 [power / power at d = { 2 diameter2 / λ }]
D/λ = 6.28
30%
40%
50%
60%
70%
80%
90%
100%
Illumination Efficiency
10 log10 [distance d in front of center of antenna / { 2 diameter2 / λ }]


10 log10 [power / power at d = { 2 diameter2 / λ }]
D/λ = 3.14
30%
40%
50%
60%
70%
80%
90%
100%
Illumination Efficiency
10 log10 [distance d in front of center of antenna / { 2 diameter2 / λ }]


10 log10 [power / power at d = { 2 diameter2 / λ }]
D/λ = 1.57
30%
40%
50%
60%
70%
80%
90%
100%
Illumination Efficiency
10 log10 [distance d in front of center of antenna / { 2 diameter2 / λ }]


10 log10 [power / power at d = { 2 diameter2 / λ }]
D/λ = 0.785
30%
40%
50%
60%
70%
80%
90%
100%
Illumination Efficiency
10 log10 [distance d in front of center of antenna / { 2 diameter2 / λ }]

Square ("Panel") Antennas

	Maximum Power Density (dB relative to power density at [$d / (2 W^2 / \lambda)$] = 1)															
Width (ft)	η = 100 %	95%	90%	85%	80%	75%	70%	65%	60%	55%	50%	45%	40%	35%	30%	25%
12	11.3	12.0	12.4	12.8	13.1	13.7	14.4	15.1	15.9	16.7	17.6	18.6	19.7	20.9	22.3	23.9
6	11.3	12.0	12.4	12.8	13.1	13.7	14.4	15.1	15.9	16.7	17.6	18.6	19.7	20.9	22.3	23.9
3	11.3	12.0	12.4	12.8	13.1	13.7	14.4	15.1	15.9	16.7	17.6	18.6	19.7	20.9	22.3	23.9
2	11.3	12.0	12.4	12.8	13.2	13.7	14.4	15.1	15.9	16.7	17.6	18.6	19.7	20.9	22.3	23.9
1	11.2	12.0	12.4	12.8	13.2	13.7	14.4	15.2	15.9	16.8	17.7	18.7	19.8	21.0	22.4	24.1
0.5	11.1	12.0	12.4	12.8	13.3	13.7	14.5	15.3	16.1	17.0	17.9	19.0	20.1	21.4	22.9	24.7
0.25	10.8	11.9	12.5	13.0	13.6	14.3	15.2	16.1	17.1	18.0	19.0	19.9	21.0	22.1	23.3	24.6
0.125	9.8	11.7	12.4	13.1	13.7	14.3	14.9	15.5	16.1	16.7	17.4	18.1	18.9	19.7	20.7	21.9

	Width (ft)					
W / λ	5.788 GHz	6.175 GHz	6.7 GHz	11.2 GHz	18.7 GHz	22.4 GHz
75.3	12.8	12.0	11.1	6.62	3.96	3.31
37.7	6.40	6.00	5.53	3.31	1.98	1.65
18.8	3.20	3.00	2.76	1.65	0.991	0.827
12.6	2.13	2.00	1.84	1.10	0.660	0.551
6.28	1.07	1.00	0.922	0.551	0.330	0.276
3.14	0.533	0.500	0.461	0.276	0.165	0.138
1.57	0.267	0.250	0.230	0.138	0.0826	0.0689
0.785	0.133	0.125	0.115	0.0689	0.0413	0.0345

	$2 W^2 / \lambda$ (ft)					
W / λ	5.788 GHz	6.175 GHz	6.7 GHz	11.2 GHz	18.7 GHz	22.4 GHz
75.3	1929	1808	1666	997	597	498
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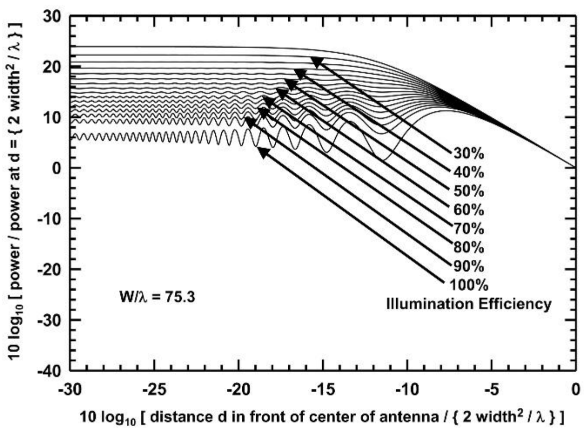
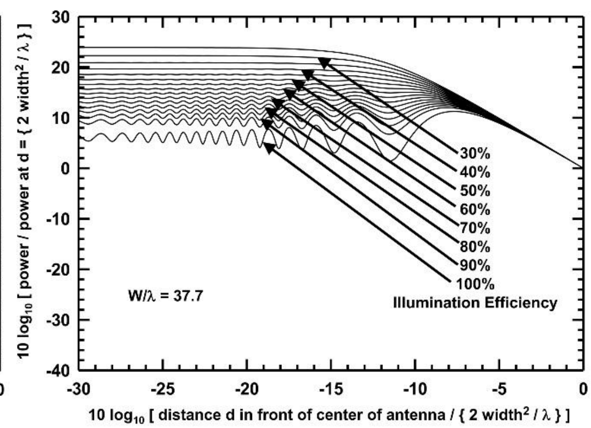
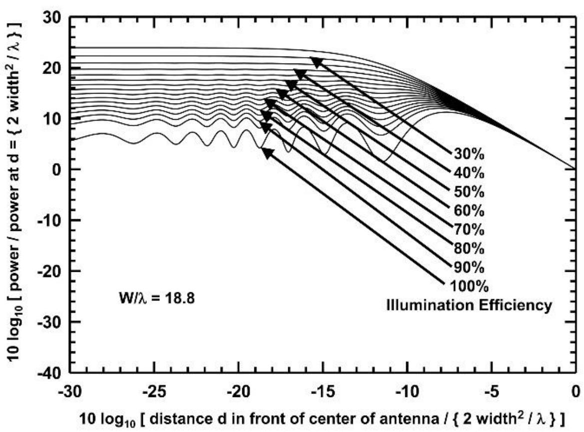
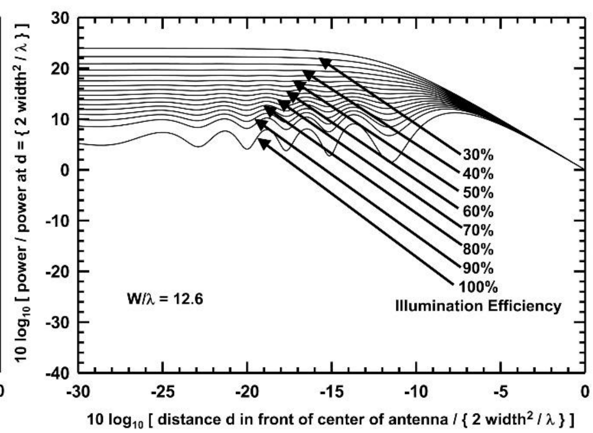
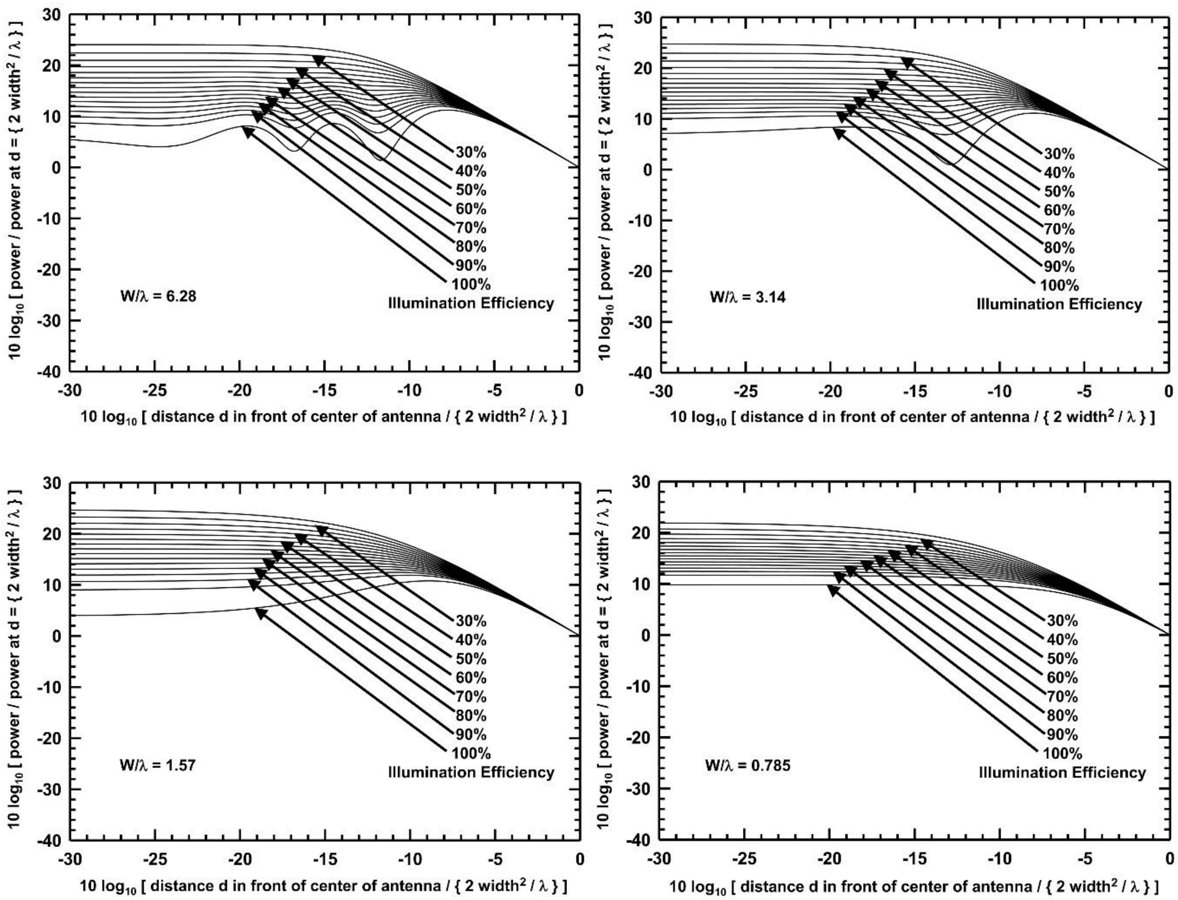
12.6	53.6	50.2	46.3	27.7	16.6	13.8
6.28	13.4	12.6	11.6	6.92	4.15	3.46
3.14	3.35	3.14	2.89	1.73	1.04	0.865
1.57	0.837	0.785	0.723	0.433	0.259	0.216
0.785	0.209	0.196	0.181	0.108	0.0648	0.0541










10 log10 [power / power at d = { 2 width2 / λ }]
10 log10 [distance d in front of center of antenna / { 2 width2 / λ }]
30
20
10
0
-10
-20
-30
-40
-30
-25
-20
-15
-10
-5
0
30%
40%
50%
60%
70%
80%
90%
100%
Illumination Efficiency
W/λ = 6.28
W/λ = 3.14
W/λ = 1.57
W/λ = 0.785